

THE APPLICABILITY OF THE EQUIVALENCE THEOREM IN χPT

Antonio DOBADO ¹ and José Ramón PELAEZ ²

Departamento de Física Teórica
Universidad Complutense de Madrid
28040 Madrid, Spain

and

María Teresa URDIALES ³

Departamento de Física Teórica
Universidad Autónoma de Madrid
28049 Madrid, Spain

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Abstract

We have explicitly calculated the tree level elastic scattering cross sections of two longitudinal gauge bosons, up to four derivatives in the chiral expansion both with and without using the Equivalence Theorem (ET). The numerical results show the existence of new and severe restrictions in the ET energy applicability range, as it was stated in our recent derivation, which we also review here, of the precise ET version in the Chiral Lagrangian description of the Standard Model Symmetry Breaking Sector.

¹E-mail: dobado@cernvm.cern.ch

²E-mail: pelaez@fis.ucm.es

³E-mail: mayte@ccuam3.sdi.uam.es

1 Introduction

In this paper we try to clarify the problem of the applicability of the so-called Equivalence Theorem (ET) [1, 2, 3] which relates, at high energies, the longitudinal electroweak gauge bosons S matrix elements with those elements where these gauge bosons have been replaced by their corresponding would be Goldstone Bosons (GB). This relation is very useful to obtain information from the future LHC data about the Standard Model (SM) Symmetry Breaking Sector (SBS), since computations are much easier to do for scalars than for longitudinal gauge bosons.

Despite the very precise data collected at LEP we almost have no information on the SMSBS and we do not know, at present, which is the dynamics responsible for the spontaneous breaking of the electroweak group $SU(2)_L \times U(1)_Y$ to the electromagnetic group $U(1)_{em}$, so it would be interesting to develop a model independent framework to describe phenomenologically the SBS mechanism. Recently this approach has been followed borrowing a formalism from low-energy hadron physics that is called Chiral Perturbation Theory (χ PT) [4, 5], and it has been proved to be also quite useful for the analysis of the precision measurements obtained at LEP [6].

In order to apply χ PT to the SMSBS description one assumes that there must be some physical system coupled to the SM with a global symmetry breaking from a gauge group G to another gauge group H producing the spontaneous symmetry breaking of $SU(2)_L \times U(1)_Y$ to $U(1)_{em}$ which yields the W^\pm and Z masses through the standard Higgs mechanism. The GB related to the global G to H symmetry breaking are nothing but coordinates in the coset space G/H and their low energy dynamics is described by a gauged Non-Linear Sigma Model plus an infinite number of higher derivative terms (but finite for practical purposes) needed for the renormalization of the model.

As we have said before both χ PT and the ET have been used together to describe the scattering of gauge bosons longitudinal components, even though a rigorous proof of this theorem in the Chiral Lagrangian description of the SM has only been presented very recently [7], and although it is known that correction factors have to be taken into account due to the different renormalization of the GB and the gauge bosons even in the simple original formulation of the Minimal Standard Model (MSM) [8], as well as in the Effective Lagrangian formalism [9].

In this paper we follow the proof of the precise statement of the theorem valid for a chiral lagrangian description of the SMSBS including the renormalization effects, which was given in [7]. That derivation is based on the nice formal proof of the ET for the MSM by Chanowitz and Gaillard [2], and consists in obtaining Ward-Slavnov Taylor identities from the BRS symmetry [10] of the model and then to translate these relations between Green functions, to S matrix elements but we take into account the peculiarities of χ PT and include renormalization factors. To implement the BRS symmetry we could have followed the standard Faddeev-Popov quantization procedure, but since we want a proof valid for any GB parametrization choice, we will use a more general method given in [11] that deals more elegantly with non-linear gauge fixing

conditions which will be needed to ensure the covariance of the quantum Lagrangian under coordinate changes in the GB coset.

Once we have obtained the precise formulation of the ET in the Chiral Lagrangian formalism, we will discuss its energy applicability range coming from the different approximations that we need in its derivation. We will also show some numerical results for the elastic scattering amplitude of two longitudinal gauge bosons, which will allow us to compare the tree level computations of the chiral lagrangian up to four derivatives , both with or without using the ET, and therefore to check the applicability window obtained in the proof presented in [7]

2 The chiral lagrangian formalism and the SMSBS

The known facts on the SMSBS impose some conditions on G and H :

- a) $\dim K = \dim G/H = 3$ since we need three GB to give mass to W^\pm and Z .
- b) G contains $SU(2)_L \times U(1)_Y$ so that the symmetry breaking sector couples to the electroweak gauge bosons.
- c) H contains the custodial group $SU(2)_{L+R}$ in order to ensure the experimental relation $\rho \simeq 1$ [12]. This constraint yields $\rho = 1$ once the gauge couplings are set to zero and implies that the photon is massless since $U(1)_{em}$ is contained in $SU(2)_{L+R}$.

It has been shown in [7] that these conditions completely determine the G and H groups to be $G = SU(2)_R \times SU(2)_L$ and $H = SU(2)_{L+R}$ and thus $K = G/H = S^3$. Therefore, we can describe the SMSBS in the Chiral Lagrangian formalism as a gauged non-linear sigma model based on the coset space $K = G/H = SU(2)_L \times SU(2)_R / SU(2)_{L+R} = S^3$ with gauge group $SU(2)_L \times U(1)_Y$. Thus we can write the lagrangian

$$\begin{aligned} \mathcal{L}_g &= \mathcal{L}_{YM}^L + \mathcal{L}_{YM}^Y + \frac{1}{2} g_{\alpha\beta}(\omega) D_\mu \omega^\alpha D^\mu \omega^\beta \\ &+ \text{higher covariant derivative terms} \end{aligned} \quad (1)$$

where \mathcal{L}_{YM}^L and \mathcal{L}_{YM}^Y are the usual Yang-Mills lagrangians for the $SU(2)_L$ and $U(1)_Y$ gauge fields W_μ^a and B_μ ; the ω^α fields are arbitrary coordinates on the coset S^3 chosen so that for the classical vacuum $\omega^\alpha = 0$. The non-linear transformation of the GB $\omega^\alpha(x)$ under the action of an infinitesimal G element defines the killing vectors ξ_a^α through $\delta\omega^\alpha = \xi_a^\alpha(\omega)$. Notice that the a index runs from 1 to 6 where the values 1 to 3 correspond to the unbroken $H = SU(2)_L \times SU(2)_R$ generators. Now we can build the S^3 metrics $g_{\alpha\beta}$ through the dreibein $e_a = e_a^\alpha \partial / \partial \omega^\alpha$ with $e_a^\alpha = \xi_{a+3}^\alpha$ for $a = 1, 2, 3$ which is nothing but the set of killing vectors corresponding to the 3 broken generators. We define the metrics as $g^{\alpha\beta} = e_a^\alpha e^{\beta a}$. This procedure ensures that G is the isometry group of S^3 with that metrics. The covariant derivatives are defined as:

$$D_\mu \omega^\alpha = \partial_\mu \omega^\alpha - g l_a^\alpha W_\mu^a - g' y^\alpha B_\mu \quad (2)$$

where l_a^α and y^α are the killing vectors corresponding to the gauge groups $SU(2)_L$ and $U(1)_Y$ respectively. The *higher derivative terms* include any covariant (in the space-time and the S^3 sense) piece containing a bigger number of covariant derivatives with arbitrary couplings so that we can reproduce any dynamics compatible with the $SU(2)_L \times SU(2)_R / SU(2)_{L+R} = S^3$ to $SU(2)_L \times U(1)_Y$ symmetry breaking. Thus the gauge transformations are:

$$\begin{aligned}\delta\omega^\alpha &= l_a^\alpha \epsilon_L^a(x) + y^\alpha \epsilon_Y(x) \\ \delta W_\mu^a &= \frac{1}{g} \partial_\mu \epsilon_L^a(x) + \epsilon_{abc} W_\mu^b \epsilon_{Lc}(x) \\ \delta B_\mu &= \frac{1}{g'} \partial_\mu \epsilon_Y(x)\end{aligned}\tag{3}$$

3 The quantum lagrangian and BRS invariance

The formal proofs of the ET are based in the BRS symmetry of the quantized lagrangian [2, 3] and they are performed in t'Hooft or renormalizable (R_ξ) gauges. When dealing with the Chiral Lagrangian formalism, the usual linear choice for the t'Hooft gauge fixing function does not yield a covariant lagrangian under reparametrizations of the GB coset, due to the fact that the GB fields are coordinates, and their contraction does not transform properly. In the χPT applications many different GB parametrizations are commonly used, that is why we are interested in an ET proof valid for any coordinate choice, and therefore, if we want to use a t'Hooft gauge fixing function, the dependence on the GB fields should be nonlinear. However, it is well known that nonlinear gauge fixing conditions lead to appearance of quartic ghost interactions even when they were not present in the original lagrangian. That is why we are going now to quantize the model built in the preceding section following a different procedure [11] which deals more elegantly with these nonlinear gauge fixing conditions and quartic ghost interactions.

The above gauge transformations satisfy the Jacobi identity as well as closure relations (see [7] for details) and therefore we can build the corresponding nilpotent (\bar{s})- s (anti)-BRS transformations by introducing the anti-commuting ghost fields c_a and \bar{c}_a , and the commuting auxiliary field B_a with $a = 1, 2, 3, 4$.

For further convenience we will unify the notation so that the first three values of the gauge indices $a = 1, 2, 3$ refer to the $SU(2)_L$ group and $a = 4$ refers to $U(1)_Y$, thus the gauge field W_μ^a with $a = 1, 2, 3, 4$ will be defined as $W_\mu^a = W_\mu^a$ for $a = 1, 2, 3$ and $W_\mu^4 = B_\mu$. In addition we introduce the Killing vector L_a^α with $a = 1, 2, 3, 4$ as $L_a^\alpha = gl_a^\alpha$ for $a = 1, 2, 3$ and $L_4^\alpha = g'y^\alpha$ and the completely antisymmetric symbols f_{abc} with $a = 1, 2, 3, 4$ as $f_{abc} = g\epsilon_{abc}$ for $a = 1, 2, 3$ and $f_{ab4} = 0$.

The nilpotency properties (which are equivalent to the Jacobi and closure relations) $s^2 =$

$s\bar{s} + \bar{s}s = \bar{s}^2 = 0$ allow us to define a (anti)-BRS invariant quantum lagrangian as follows:

$$\mathcal{L}_Q = \mathcal{L}_g + \frac{1}{2}s\bar{s}[W_\mu^a W^{\mu a} + 2\xi f(\omega) + \xi c^a \bar{c}_a] \quad (4)$$

where f is any scalar analytical function with $\partial f(\omega)/\partial \omega^\alpha = \omega^\alpha + O(\omega^2)$. The new terms added to the lagrangian (see [7] for details) are a generalization of the Faddeev-Popov terms in a t'Hooft like gauges, which have two main advantages: First, they provide us with a well defined R_ξ propagator to be used in perturbation theory. Second, they cancel the GB and gauge boson mixing terms in the lagrangian. In addition, this generalized method produces other GB-gauge boson and ghost-gauge boson interactions. For gauges different from that of Landau ($\xi = 0$) we also have quartic ghost interactions and GB-ghosts interactions.

Once we have a (anti)-BRS invariant lagrangian we can derive, using the standard functional methods, the Ward-Slavnov-Taylor identities for dimensionally regularized Green functions. It is worth mentioning that we use dimensional regularization not only to preserve the (anti)-BRS invariance but also to avoid the $-\frac{i}{2}\delta^n(0)tr \log g$ term that appears in the quantum lagrangian of the non-linear sigma model (NLSM) coming from the path integral measure of the GB fields [13].

As a matter of fact, and in order to make physical predictions, we are interested in renormalized Green functions. Therefore we have to consider the renormalized lagrangian which consists on that of eq.4 plus other terms with the corresponding couplings needed to reproduce all the divergent structures which appear in the Green functions. At present, the form of these counterterms is known only up to four derivatives [14], but they should also be (anti)-BRS invariant, since if this was not the case, the gauge invariance of the model would be anomalous, i.e., broken by quantum effects. Nevertheless, it is well known that even though we had chiral fermions coupled to GB and gauge bosons, the SM hypercharge assignments are such that all possible gauge and mixed gauge-gravitational anomalies, including the non-perturbative $SU(2)$ discovered by Witten [15], do cancel when the number of colors is $N_c = 3$. As we are interested in reparametrization invariance we should also worry about the potential anomalies that could break the invariance under coordinate changes in coset, but these are absent since our NLSM is defined in a space of lower dimension than space-time, as it was shown in [16]. Recently some papers have appeared where the applicability of the ET is discussed in models where some anomalies are present, but this is not our case and we will only refer the reader to the literature [17].

Once we have taken into account all these considerations, we obtain a (anti)-BRS invariant lagrangian with infinite terms which can be understood as the renormalized lagrangian for a theory with infinite couplings written in terms of the bare quantities. However, we can also use the renormalized fields and couplings to write this lagrangian, so that all the terms keep same form (as the theory is renormalizable in the generalized sense described above) although they are multiplied by renormalization Z factors. The renormalized and the bare fields (denoted with a 0 subscript) and gauge couplings are related as follows:

$$\begin{aligned}
W_{0\mu}^a(x) &= Z_3^{(a)1/2} W_\mu^a(x); \omega_o^\alpha(x) = Z_\omega^{(\alpha)1/2} \omega^\alpha(x); g_0^{(a)} = Z_g^{(a)} g^{(a)}; \xi_0^{(a)} = Z_\xi^{(a)} \xi^{(a)} \\
c_0^a(x) &= \tilde{Z}_2^{(a)1/2} c^a(x); \bar{c}_0^a(x) = \tilde{Z}_2^{(a)1/2} \bar{c}^a(x); B_0^a(x) = \tilde{Z}_2^{(a)} B^a(x); v_0 = Z_v^{1/2} v
\end{aligned} \tag{5}$$

where $g^{(a)} = g$ for $a = 1, 2, 3$ and $g^{(4)} = g'$ and from now on we use indices between parenthesis as labels which are not summed. Thanks to the gauge structure of the theory the first three Z_3 are equal. Indeed, there are infinite relations between the bare and the renormalized couplings appearing in the chiral lagrangian. It is straightforward now to obtain a set of "renormalized" (anti)-BRS transformations leaving invariant the renormalized lagrangian written in terms of the renormalized fields and couplings:

$$\begin{aligned}
s_R[\omega^\alpha] &= X^{(a)} L_{Ra}^\alpha c^a & \bar{s}_R[\omega^\alpha] &= X^{(a)} L_{Ra}^\alpha \bar{c}^a \\
s_R[W^{\mu a}] &= X^{(a)} D_{Rc}^{\mu a} c^c & \bar{s}_R[W^{\mu a}] &= X^{(a)} D_{Rc}^{\mu a} \bar{c}^c \\
s_R[c^a] &= -\frac{X^{(a)}}{2} f_{Rbc}^a c^b c^c & \bar{s}_R[c^a] &= -\frac{X^{(a)}}{2} f_{Rbc}^a \bar{c}^b c^c \\
s_R[\bar{c}^a] &= X^{(a)} \frac{B^a}{\sqrt{\xi^{(a)}}} & \bar{s}_R[\bar{c}^a] &= -X^{(a)} \left(\frac{B^a}{\sqrt{\xi^{(a)}}} + f_{Rbc}^a \bar{c}^b c^c \right) \\
s_R[B^a] &= 0 & \bar{s}_R[B^a] &= -X^{(a)} f_{Rbc}^a \bar{c}^b B^c
\end{aligned} \tag{6}$$

where $L_{Ra}^\alpha = Z_\omega^{(\alpha)-1/2} Z_3^{(a)1/2} L_a^\alpha$, $f_{Rbc}^a = Z_g^{(a)} Z_3^{(a)1/2} g f_{bc}^a$, and $X^{(a)} = \tilde{Z}_2^{(a)1/2} / Z_3^{(a)}$. One could think that the appearance of the L factors, which are nonlinear in the GB fields, will make the relation between gauge bosons and GB extremely cumbersome, since this relation will be derived from the BRS symmetry of the lagrangian. We will see that this is not the case.

These "renormalized" symmetry of the quantum lagrangian will allow us in the next section to apply the standard functional methods to obtain the corresponding Ward-Slavnov-Taylor identities for renormalized Green functions which lead us to the ET.

4 Ward-Slavnov-Taylor Identities

Our aim is to obtain the relationship between S -matrix elements involving longitudinal gauge bosons W_L and those elements where we have replaced the external W_L by GB. To that end we will first obtain, from the BRS invariance of the renormalized lagrangian, Ward-Slavnov-Taylor identities that later will be translated, using the Lehmann-Symanzik-Zimmermann (LSZ) reduction formula, in relations between S -matrix elements.

We start by remembering that the generating functional for renormalized connected Green's functions $W_R(x_1, \dots, x_n)$ is given, in momentum space, by the following definition:

$$W_R[J] = (2\pi)^4 \sum_{n=1} \int \prod_{i=1}^n \frac{d^4 p_i}{(2\pi)^4} \delta^4(\sum_i p_i) J_{i_1}(-p_1) \dots J_{i_n}(-p_n) W_{R\ i_1, \dots, i_n}(p_1, \dots, p_n) \quad (7)$$

where $W_{R\ i_1, \dots, i_n}(p_1, \dots, p_n)$ are renormalized Green functions. We can now use the BRS invariance of the lagrangian (A stands for any field appearing in the quantum lagrangian i.e. $A_i = \omega^\alpha, W_\mu^a, c^a, \bar{c}^a, B^a$) to write:

$$\sum_i \int d^4 x \langle s_R[A_i] \rangle_J J_i(x) = 0 \quad (8)$$

where we can write, in general, the BRS transformations as sums of linear or nonlinear field products as:

$$\langle s_R[A_i] \rangle_J = \sum_n s_{A_i}^{i_1 \dots i_n} \langle A_{i_1} \dots A_{i_n} \rangle_J = \sum_n s_{A_i}^{i_1 \dots i_n} \frac{\delta^{(n)} W_R[J]}{\delta J_{i_1} \dots \delta J_{i_n}} \quad (9)$$

Where, as usual, J_i is the external current corresponding to the A_i field. In the GB case the above expression corresponds to the series expansion of the nonlinear BRS transformation, for the rest of the fields there will appear terms with just one field, or in the case of $A_i = W^\mu, c^a$ also terms with a two field product. Therefore the BRS invariance condition can be written as:

$$I[J] = \sum_i \sum_n s_{A_i}^{i_1 \dots i_n} \int \frac{d^4 q d^4 k_1 \dots d^4 k_{n-1}}{(2\pi)^{4n}} \frac{\delta^{(n)} W_R[J]}{\delta J_{i_1}(q - k_1) \dots \delta J_{i_n}(k_{n-1})} J_i(-q) = 0 \quad (10)$$

It is now straightforward to obtain Ward-Slavnov-Taylor identities by taking functional derivatives with respect to $J_i(p)$ at $J = 0$. Indeed, we are interested on identities involving the B field, which is nothing but the gauge fixing condition that intuitively identifies W_L and the GB. Therefore we write:

$$\left. \frac{\delta}{\delta J_{\bar{c}_{a_1}}(-k)} \prod_{j=2}^s \frac{\delta}{\delta J_{B_{a_j}}(-k_j)} \prod_{k=1}^m \frac{\delta}{\delta J_{A_k}(-p_k)} I[J] \right|_{J=0} = 0 \quad (11)$$

Where we will impose that the currents J_{A_k} are only associated to physical A_k fields. Taking just two functional derivatives we obtain the following relation between the two leg Green functions (see [7] for details):

$$\frac{X^{(b)}}{\sqrt{\xi^{(b)}}} W_{B^b l}(p) = -X^{(a)} D_{Rl}^a(p) W_{c^a \bar{c}^b}(p) \quad (12)$$

where:

$$D_{Rl}^a(p) = i p_\mu (1 + \Delta_3(p^2)) \delta_l^{W_{\mu a}} + (L_{Ra}^{(0)\alpha} + \Delta_{2a}^\alpha(p^2)) \delta_l^{\omega^\alpha} \quad (13)$$

and we have used:

$$\begin{aligned}
ip_\mu \Delta_3(p^2) &= f_{Rdc}^a W_{c^a \bar{c}^b}^{-1}(p) \int \frac{d^4}{(2\pi)^4} W_{W^{\mu d} c^c \bar{c}^b}(p-q, q, p) \\
\Delta_{2a}^\alpha(p^2) &= L_{Ra}^{(1)\alpha\beta} W_{c^c \bar{c}^b}^{-1}(p) \int \frac{d^4}{(2\pi)^4} W_{\omega^\beta c^c \bar{c}^b}(p-q, q, p) + \dots
\end{aligned} \tag{14}$$

Note three important features that were not present in the formal proof of [2]: a) the renormalization X factors, b) the $L_R^{(0)}$ term which is the term coming only from the linear part of the GB BRS transformation, and that will be, at the end, the only remainder of the complicated realization of the symmetry at lowest order in g or g' , thus simplifying the relation between GB and gauge bosons that one would expect naively from the nonlinear gauge fixing condition; and c) the appearance of the Δ terms that were correctly introduced in [18] for the SM, and more recently in [9] in the context of Chiral Lagrangians. It is important to remark that these Δ_2 and Δ_3 terms are of higher order in g or g' than $L_R^{(0)}$ and 1, respectively, and therefore we will neglect them when using only the lowest order in the weak couplings.

In order to obtain the general expression, we have to notice from the BRS transformations that we will not get any contribution if $A_i = B$, neither when $A_i = \omega, c$ because there are no J_ω nor J_c derivatives. As the A_k are physical, their polarization vectors will cancel the derivative term in $s_R[W_\mu^a] = ik_\mu c^a + \epsilon_{Rbc}^a W_{\mu b} c_c$ since $\epsilon \cdot k_\mu = 0$. Therefore, we only have to take into account the contributions from $s_R[\bar{c}]$ and the part which is left from $s_R[W_\mu^a]$ that will be called "bilinear terms". Thus we obtain:

$$\frac{X^{(a_1)}}{\sqrt{\xi^{(a_1)}}} W_{B_{a_1} B_{a_2} \dots B_{a_s} A_1 \dots A_m}(k_1, \dots, k_s, p_1, \dots, p_m) + \text{bilinear terms} = 0 \tag{15}$$

where $\sum_i k_i = -\sum_i p_i$. As the a_1 index is free we can drop the factor $X/\sqrt{\xi}$ which is irrelevant. However this is a relation between Green functions, and we have to apply the LSZ reduction formula to translate it to S -matrix elements:

$$\left(\prod_{i=1}^m W_{A_i A_i}(p_i) \right) \sum_{l_j} \left(\prod_{j=1}^s W_{B_{a_j} l_j}(k_j) \right) S_{l_1 \dots l_s A_1 \dots A_m}^{off-shell}(k_1 \dots k_s, p_1 \dots p_s) + \text{bilinear terms} = 0 \tag{16}$$

The next step in the LSZ procedure is to multiply the above equation by the inverse A_i propagators, and to set their momenta on-shell, that is $p_i^2 = m_{A_i}^2$, then the "bilinear terms" cancel since they contain a Green's function with at least one off-shell momentum, and therefore they do not have the pole needed to compensate for $W_{A_i A_i}^{-1}(p_1) \rightarrow 0$ when $p_1^2 = m_{A_1}^2$. We can now use eq.12 to substitute the B field two point functions and thus we get:

$$\sum_{l_j} \left(\prod_{j=1}^s \frac{\sqrt{\xi^{(a_j)}}}{X^{(a_j)}} X^{(c_j)} W_{c_j \bar{c}^{a_j}}(k_j) D_{Rl_j}^{c_j}(k_j) \right) S_{l_1..l_s A_1..A_m}^{off-shell}(k_1..k_s, p_1..p_s) \Big|_{p_i^2=m_{A_i}^2} = 0 \quad (17)$$

The $\sqrt{\xi^{(a_j)}}/X^{a_j}$ factors are again irrelevant since we have not contracted the a_j indices. We still have to multiply by the ghost inverse two point functions $W_{c_j \bar{c}^{a_j}}^{-1}(k_j)$ which are non-diagonal in principle. In so doing we see that the d_j indices are free again and we can drop the other X factors. We finally obtain:

$$\sum_{l_1..l_r} \prod_{i=1}^s D_{Rl_i}^{a_i}(p_i) S_{l_1..l_s, A_1..A_m}^{off-shell}(p_1..p_r, k_1..k_m) \Big|_{p_i^2=m_{A_i}^2} = 0 \quad (18)$$

5 The Equivalence Theorem

The last step in the LSZ formulae is to set all the momenta on-shell, but before that, we have to obtain the physical combinations out of the W_μ fields which appear in the D_R operator. That is achieved by means of a transformation $\widetilde{W}_\mu^a = R^{ab} W_\mu^b$, whose most general form is :

$$\begin{pmatrix} \widetilde{W}_\mu^1 \\ \widetilde{W}_\mu^2 \\ \widetilde{W}_\mu^3 \\ \widetilde{W}_\mu^4 \end{pmatrix} = \begin{pmatrix} W_\mu^- \\ W_\mu^+ \\ Z_\mu^{phys} \\ A_\mu^{phys} \end{pmatrix} = \begin{pmatrix} 1/\sqrt{2} & i/\sqrt{2} & 0 & 0 \\ 1/\sqrt{2} & -i/\sqrt{2} & 0 & 0 \\ 0 & 0 & \cos\theta & -\sin\theta \\ 0 & 0 & \sin\theta' & \cos\theta' \end{pmatrix} \begin{pmatrix} W_\mu^1 \\ W_\mu^2 \\ W_\mu^3 \\ W_\mu^4 \end{pmatrix} \quad (19)$$

The propagators of these new renormalized fields present poles in the right values of the corresponding physical masses. According to those definitions we also introduce: $\tilde{L}_{R\alpha}^{(0)b} = L_{R\alpha}^{(0)a} (R^{-1})^{ba}$ and $\tilde{\Delta}_i$. Therefore we obtain:

$$\sum_{l_1..l_r} \prod_{i=1}^s \widetilde{D}_{Rl_i}^{a_i}(p_i) S_{l_1..l_s, A_1..A_m}(p_1..p_r, k_1..k_m) = 0 \quad (20)$$

where now

$$\widetilde{D}_{Rl}^a(p) = ip_\mu (1 + \tilde{\Delta}_3(M_{phys}^2)) \delta_l^{\widetilde{W}_{R\mu a}} + (\tilde{L}_{Ra}^{(0)\alpha} + \tilde{\Delta}_{2a}^\alpha(M_{phys}^2)) \delta_l^{\omega_\alpha} \quad (21)$$

Notice that the p_i momenta are on-shell for the massive physical vector bosons, and since the Δ terms do not depend on the energy or external momenta. From now on we will use amplitudes instead of S matrix elements as it is customary in χPT . As a matter of fact it is more convenient

to obtain the ET from the following relation between amplitudes that we will write symbolically as:

$$\begin{aligned} \left(\prod_{i=1}^n \epsilon_{(L)\mu_i} \right) T(\widetilde{W}_{a_1}^{\mu_1}, \dots, \widetilde{W}_{a_n}^{\mu_n}; A) = \\ = \sum_{l=0}^n (-i)^l \left(\prod_{i=1}^l v_{\mu_i} \right) \left(\prod_{j=l+1}^n K_{\alpha_j}^{a_j} \right) \bar{T}(\widetilde{W}_{a_1}^{\mu_1} \dots \widetilde{W}_{a_l}^{\mu_l}, \omega_{\alpha_{l+1}} \dots \omega_{\alpha_n}; A) \end{aligned} \quad (22)$$

Where we have introduced $v_\mu = \epsilon_{(L)\mu} - p_\mu/m \simeq O(M_{phys}/E)$, we have omitted the irrelevant indices, and we have defined:

$$K_{Ra}^\alpha = \frac{\tilde{L}_{Ra}^{(0)\alpha} + \tilde{\Delta}_{2a}^\alpha (M_{phys}^{(a)2})}{M_{phys}^{(a)} (1 + \tilde{\Delta}_3 (M_{phys}^{(a)2}))} \quad (23)$$

which do not depend on the momenta, and \bar{T} which is the sum over all the amplitudes with independent permutations of fields and indices for a given l value. This relation was first obtained in [2] but without taking into account the K factors (see [7]).

In the proof by Chanowitz and Gaillard the next step was to neglect at high energies the terms containing v_μ factors since $v_\mu \simeq O(M_{phys}/E)$ and the amplitudes in the MSM satisfy the unitarity bounds and do not grow with energy. Therefore they were able to drop at high energies all terms in the RHS of eq.22 but the one with $l = 0$ which is precisely that with all external \widetilde{W}_L substituted by GB, thus obtaining the ET. However, in our case we are not allowed to do so since the amplitudes in χPT are obtained as a truncated series in the energy and we cannot simply neglect the terms containing v_μ factors, but we have to use power counting methods to obtain the leading orders. A similar problem arises when dealing with the large m_H limit of the SM [19].

The K factors in eq.22 include the renormalization effects on the ET which also appear in the Chiral Lagrangian formalism, as it has been recently shown in [9] where the authors arrive, using a different quantization procedure, to similar results to us for $g' = 0$ and the GB parametrization $U = \exp(i\sigma^a \omega^a/v)$. They concentrate in the renormalization factors which correct the ET version without performing the power counting analysis. (See also [20] for a general discussion)

When dealing with χPT we can expand the amplitudes as Laurent series in $E/4\pi v$ up to a positive power N by fixing the maximum number of derivatives in the Lagrangian. However, these amplitudes should satisfy the Low Energy Theorems (second reference in [12]) in the $M^2 \ll E^2$ regime, so that the energy negative powers can be written as $(M/E)^k$. Therefore:

$$\bar{T}(\widetilde{W}_{a_1}^{\mu_1} \dots \widetilde{W}_{a_l}^{\mu_l}, \omega_{\alpha_{l+1}} \dots \omega_{\alpha_n}; A) \simeq \sum_{k=0}^N a_l^k \left(\frac{E}{4\pi v} \right)^k + \sum_{k=1}^{\infty} a_l^{-k} \left(\frac{M}{E} \right)^k \quad (24)$$

Notice that in order to simplify the analysis we will set momentarily $g' = 0$ and that we have omitted the field indices in a_l^k . The coefficients in this formulae, which can contain logarithms of the energy and therefore should be understood formally, can be expanded perturbatively on g , for instance: $a_l^h = a_{lL}^h(1 + O(g/4\pi))$ where a_{lL}^h is the lowest order term in the g expansion. In most renormalization schemes we have $M \simeq M_{phys}(1 + O(g/4\pi))$ and therefore we can write $K_\alpha^a \simeq K_\alpha^{a(0)} + K_\alpha^{a(1)}(g/4\pi) + \dots$ where now these coefficients are energy independent. Once we introduce these expansions in eq.22, if we neglect the order $O(M/E)$ and $O(E/4\pi v)^{h+1}$ terms, we obtain:

$$\begin{aligned} & \left(\prod_{i=1}^n \epsilon_{(L)\mu_i} \right) T(\widetilde{W}_{a_1}^{\mu_1}, \dots, \widetilde{W}_{a_n}^{\mu_n}; A) \simeq \\ & \simeq \left(\prod_{j=1}^n K_{\alpha_j}^{a_j(0)} \right) \sum_{k=0}^h (a_{0L}^k (1 + O(g/4\pi))) \left(\frac{E}{4\pi v} \right)^k + O\left(\frac{M}{E}\right) + O\left(\frac{E}{4\pi v}\right)^{h+1} \end{aligned} \quad (25)$$

which is the precise formulation of the ET in the Chiral Lagrangian formalism (for the sake of brevity the indices α of a_0).

It is this expression the one which will allow us to analyze the applicability of the ET in the χPT description of the SMSBS, since if we want these approximations to make sense, we are only left with the following applicability window:

$$\begin{aligned} M &\ll E \ll 4\pi v \\ g/4\pi &\ll (E/4\pi v)^{h+1} \end{aligned} \quad (26)$$

The first inequality was present in the original formulation of the ET, and it comes from neglecting the $O(M/E)$ contributions. The second inequality is characteristic of χPT and is due to the fact that in the Chiral Lagrangian formalism we always obtain the amplitudes as truncated series in the energy, neglecting the $O(E/4\pi v)^{h+1}$ term. If we want to obtain sensible physical predictions, definitely we cannot trust the chiral expansion beyond $E = 4\pi v$ (Usually much before). It is then expected that the amplitude in eq.25 calculated with and without the ET would yield identical results beyond that energy limit, but it will not have any physical meaning. The last constraint comes from neglecting the $O(M/E)$ term while keeping at the same time the $O(E/4\pi v)^h$ contribution, since we expect the former to be much smaller than the latter.

It is straightforward now to generalize the preceding results to $g' \neq 0$ because $g' \ll g$ as well as $M_Z^{phys} \simeq M_W^{phys} \simeq M^{(a)}$ for any a (all the different masses are of the same order when counting energy powers). We only have to take into account the lowest order of the a coefficients in the g or g' expansion so that the same reasoning we had used when $g' = 0$ is still valid.

6 Some numerical results

In order to check the validity of the ET for chiral lagrangians obtained in the previous section we have explicitly computed the tree level cross sections (so that all Z factors are equal to one) up to four derivatives obtained from the chiral lagrangian considered in the second reference in [14] for the processes, $Z_L^0 Z_L^0 \rightarrow Z_L^0 Z_L^0$ and $W_L^+ W_L^- \rightarrow Z_L^0 Z_L^0$ and $W_L^\pm Z_L^0 \rightarrow W_L^\pm Z_L^0$. We have used chiral coordinates for the parametrization of the coset space S^3 , i.e. we have grouped the GB fields in a $SU(2)$ matrix field as $U(x) = \exp(i\omega^a \sigma^a / v)$ and we have worked in the Landau gauge. The computation was done in two ways; first we have calculated the amplitude for the corresponding gauge bosons and then we have projected them into their longitudinal components. Once we had the S matrix elements we have computed the cross sections which we have compared with those obtained using the ET, that is, only with external GB. It is important to remark that the ET as stated in eq.25 only allows us to use the lowest order in g or g' , and therefore, in this case, we have only taken into account GB internal lines. For the sake of definiteness we have considered two different models. The first one corresponds to a selection of the four derivative couplings that reproduces the low energy behaviour of the MSM. Thus the couplings become a function of the Higgs mass which is taken to be equal to $1TeV$. In the second model we select the values of the couplings so that they correspond to a QCD-like theory with $N_C = 3$ (in the large N_c limit).

Some preliminary results obtained after our numerical computations are shown in the figures. Figs.1 and 2 display the comparison between the high energy behavior of the GB and the longitudinal components of the gauge bosons cross sections for the MSM and QCD models respectively. As it can be seen, a perfect agreement is found between both cross sections in the three studied channels. However, as it was commented in the previous section, this agreement does not mean at all that those cross sections reproduce properly the underlying physics (the MSM or the QCD model in this case), since we have been taking into account only up to four derivative terms in our computations. That is the reason why the $4\pi v$ energy upper bound is set in eq.26 for the validity of the standard χPT computations. The same happens in the χPT description of the low-energy pion interactions where it is well known that the standard four derivatives computation only describes properly the experimental data for energies well below $4\pi f_\pi$.

However, from the considerations done in the previous section about the applicability window of the ET, we do not expect such a good agreement in the low-energy regime, let us say below $\simeq 1.5TeV$, since higher orders in g or g' may be not negligible. This fact is confirmed in fig.3 and fig.4 where we plot the same processes considered in fig.1 and fig.2 but concentrating the attention in the low-energy region ($\sqrt{s} \leq 1.5TeV$). In this regime, we can see that the cross sections obtained using the ET (and therefore, at lowest order in the weak couplings), do not reproduce well those coming from the complete tree level computations. However, the discrepancies are only due to higher g and g' orders, since we have performed the same comparison

in the $g, g' \rightarrow 0$ limit, and the curves obtained with the two procedures overlap. This check confirms the ET statement of eq.25, but it seems that the corrections due to the weak couplings are too relevant in this low-energy regime to be neglected. One could think that the amplitudes obtained using the ET as in eq.25 would improve if we add the tree level diagrams to the next order in g and g' with four external GB fields but also internal gauge bosons. However, from the proof of the ET that we have sketched here we can see that in order to reproduce the next order in the weak couplings for the W -scattering amplitudes, not only should we have considered the diagrams with internal gauge boson lines, but also those terms in eq.22 with v_μ factors, as well as contributions from the Δ terms from the K correction factors. In fact, we have checked numerically that this naive approximation does not render any relevant improvement.

In fig.3 and fig.4 we observe that for the $Z_L^0 Z_L^0 \rightarrow Z_L^0 Z_L^0$ and $W_L^+ W_L^- \rightarrow Z_L^0 Z_L^0$ the ET predictions for the scattering cross sections are more or less accurate but this is not the case at all in the $W_L^\pm Z_L^0 \rightarrow W_L^\pm Z_L^0$ channel for both models. Therefore, the numerical results seem to confirm our expectations on the impossibility of a sensible simultaneous application of the standard χPT and the ET. In spite of what our plots for the MSM and the QCD model seem to suggest, the disagreement not only appears in the $W_L^\pm Z_L^0 \rightarrow W_L^\pm Z_L^0$ channel. We have also made computations with other arbitrary coupling choices of the four derivative terms in the chiral lagrangian and we have found disagreements between the direct computation and the ET predictions in the $W_L^+ W_L^- \rightarrow Z_L^0 Z_L^0$ channel.

7 Discussion and conclusions

We want now to remark the general applicability features of the ET as it is stated in eq.25. We have derived that expression inside the Chiral Lagrangian formalism through the use of Ward-Slavnov-Taylor identities derived from the BRS invariance of the quantum lagrangian of a gauged NLSM. The fact that we have used a formalism which is covariant under reparametrizations of the GB allows us to apply the results to any coordinate choice, simply by changing the scalar f function in eq.4 which determines the actual form of the R_ξ -gauge. Since we have included the renormalization effects, eq.25 is ready to be used beyond the tree level.

However, and due to the restrictions imposed in the energy by the applicability window of eq.26, we could still ask about the real utility of the ET, which shows the high energy relation between the GB and the W_L 's S -matrix elements, in the context of the χPT description of the SMSBS, which is nothing but a low-energy description of the GB dynamics.

Let us give an example: for the important case of two longitudinal gauge bosons elastic scattering, the ET applicability range obtained from eq.26 would be $1.7TeV \ll E \ll 4\pi v \simeq 3.1TeV$, when we include in the lagrangian the terms up to four derivatives. To confirm this fact we have carefully compared the result of a direct computation of the longitudinal components of the gauge bosons from the four derivative chiral lagrangian at the tree level with the corre-

sponding ET predictions and we have shown the results in the previous section. As expected we have found that the ET, works properly as stated in eq.25, that is, at lowest order in g or g' . Unfortunately the description thus obtained does not reproduce properly the physics below 1.7TeV , since higher corrections in the weak couplings become relevant. Besides, it is well known that at high energies such as 3TeV the χPT calculations in the four derivatives approximation cannot be trusted in many cases.

It seems then that the second constraint in eq.26, which is a lower energy limit, is much stronger than the first, since it can exclude the energy region where χPT works better. Even more, the restrictions that this constraint produces on the energy applicability window get more and more severe when the calculation is done for higher loops, or what it is the same, for higher derivative terms in the Chiral Lagrangian.

It is important to remember that this discussion in terms of energy expansions, is due to the fact that at high energies the effective lagrangian does not yield a good unitary behavior for the truncated amplitudes. This fact does not allow us to simply neglect the $v_\mu = O(M/E)$ factors when extracting the leading energy term in eq.22, since the amplitudes can contain positive powers of E . Nevertheless, there are non-perturbative methods to implement unitarity in the χPT amplitudes so that at high energies they will never grow with a power of E , and therefore we are allowed to directly drop the terms with v_μ factors, thus obtaining:

$$\left(\prod_{i=1}^n \epsilon_{(L)\mu_i} \right) T(\widetilde{W}_{a_1}^{\mu_1}, \dots, \widetilde{W}_{a_n}^{\mu_n}; A) \simeq \left(\prod_{j=1}^n K_{\alpha_j}^{a_j} \right) T(\omega_{\alpha_1} \dots \omega_{\alpha_n}; A) + O(M/E) \quad (27)$$

which is the usual formal statement of the ET. This unitarization procedures include the use of Padé approximants and dispersion relations [21], large N-limit [22], etc..., and they can enlarge considerably the ET applicability range. These two methods have been shown to work very well in hadron physics, where they are even appropriate to deal with resonances, and are expected to do so in the effective lagrangian description of the SMSBS.

To conclude we want to remark that there are three different ways to apply the ET: First we find the case of a renormalizable theory as, for example, the MSM, whose amplitudes present a good high energy behavior and thus we arrive to the ET as stated in eq.27 with just a lower energy bound. As a matter of fact the only problem is the computation of the GB amplitudes and the K factors in the chosen renormalization scheme. The second possible scenario is when we use standard χPT to describe the SMSBS since now the amplitudes are truncated series in the energy. As we have already discussed, the precise statement of the theorem is that of eq.25 although it is only applicable in the energy range of eq.26. This version of the theorem is weaker, but in this case the computation of the K factors is not so hard as in the previous one since we only need to know the lowest order in their g and g' perturbative expansion. The fact that the ET only holds in the effective lagrangian formalism to the lowest order in g had already been suggested in [18]. However, the numerical results shown in the previous section

seem to indicate that probably there is no energy applicability window for this case. Finally if we describe the SMSBS by means of unitarized χPT the version of the ET is again that of eq.27 without an upper energy applicability bound but including the K factors.

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9 Figure Captions

Figure 1. We display the three channels $Z_L^0 Z_L^0 \rightarrow Z_L^0 Z_L^0, W_L^+ W_L^- \rightarrow Z_L^0 Z_L^0$ and $W_L^\pm Z_L^0 \rightarrow W_L^\pm Z_L^0$ in the high energy regime for the MSM. The continuous lines correspond to the cross sections with external GB and the dashed lines those with external gauge boson longitudinal components

Figure 2. Now we show the three channels $Z_L^0 Z_L^0 \rightarrow Z_L^0 Z_L^0, W_L^+ W_L^- \rightarrow Z_L^0 Z_L^0$ and $W_L^\pm Z_L^0 \rightarrow W_L^\pm Z_L^0$ in the high energy regime for the QCD-like model. The continuous lines correspond to the cross sections with external GB and the dashed lines those with external gauge boson longitudinal components

Figure 3. Here are shown the three channels $Z_L^0 Z_L^0 \rightarrow Z_L^0 Z_L^0, W_L^+ W_L^- \rightarrow Z_L^0 Z_L^0$ and $W_L^\pm Z_L^0 \rightarrow W_L^\pm Z_L^0$ in the low energy regime for the MSM. The continuous lines correspond to the cross sections with external GB and the dashed lines those with external gauge boson longitudinal components

Figure 4. Again we display the three channels $Z_L^0 Z_L^0 \rightarrow Z_L^0 Z_L^0, W_L^+ W_L^- \rightarrow Z_L^0 Z_L^0$ and $W_L^\pm Z_L^0 \rightarrow W_L^\pm Z_L^0$ but in the low energy regime for the QCD-like model. The continuous lines correspond to the cross sections with external GB and the dashed lines those with external gauge boson longitudinal components

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